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Predicting Growth Functions for *Pinus thunbergii* Windbreak Stands in Korea

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Abstract: *Pinus thunbergii* has widely been distributed, and is one of the main important forest resources for windbreak stands in Korea. Diameter and height growth patterns were estimated using non-linear algebraic difference equation, which requires two-measurement times T_1 and T_2. In results, of the algebraic difference equations applied, the Schumacher polymorphic equations for diameter and height showed the higher precision of the fitting. Ninety-five percent of the observations that are used to fit height model could be predicted within ±1.4 m of the actual values. Polymorphic site index curves, hence, which reflect different shapes for the different site index classes, were derived from the Schumacher equation.

1 Introduction

Based on the sophisticated technology accumulated and experienced through the implementation of various tideland reclamation projects in several decades in Korea. Among them, a master plan for 'Saemangeum' Tideland Comprehensive Reclamation Project to meet the demand of land and water resources to be required for the development of harbors and industrial complex in the years of 2000s when the western coastal era comes.

This project will cost US$ 1,806 millions (1,300 billion won) for 14 years of the implementation period. The major development works will comprise 34 ㎞ long sea-dike construction and 40,100 *ha* of new land reclamation with the additional construction works of two sluice gates, 16 pumping stations, necessary bridges, the networks of roads and canals of irrigation and drainage.

On completion of this project, industrial complex, agricultural regions for horticultural, livestock, etc., fishery ponds, settlement areas, and tourist resorts will be built up in the 28,300 *ha* of reclaimed tideland.

Reclamation work of 'Saemangeum' in Korea will be required windbreak stands. Main species for windbreak stands in Korea can be classified into 2 groups, namely coniferous and broadleaved trees. The coniferous trees are such as *Pinus densiflora*, *Pinus thunbergii*, *Cryptomeria japonica*, and *Larix leptolepis* etc.. The broadleaved trees consist of *Camellia japonica*, *Zelkova serrata*, *Alnus japonica*, and *Dendropanax morbifera* etc.. Among them, *Pinus thunbergii* has been regarded as an important species for windbreak stands in Korea. Growth of the trees by age follows sigmoid-shaped curves, and deriving growth function to presume this growth pattern supplies efficient utilization of forest resources and basis for management. The clue to successful timber management is a proper understanding of growth processes, and for this various growth functions and models have been used. Moreover, suitable method utilizing the data from two successive measurements is the algebraic difference form of a growth function that has been used by number of researchers (Clutter *et al.*, 1983; Borders *et al.*, 1984; Lee, 1998, 2000). It usually starts with the basic models which is the form of $Y_2 = f\ (Y_1, T_1, T_2)$ where the response variable Y_2 measured at time T_2 is described as a function of the same variable measured at initial time T_1 and a measure of elapsed time as a function of T_1 and T_2. The Variable Y could be basal area, top height and stems per hectare or any stand variables.

The objectives of this study, therefore, are to construct diameter, height growth equations and site index curves using the difference equation method, and to provide basic growth information for windbreak stands, which will be an essential task when the 'Saemangeum' reclamation work is successfully finished.

2 Materials and Methods

Data for this study came from *Pinus thunbergii* temporary plots grown in the middle - west coast, Korea. All of 20 plots, which were 20m x 20m size each plot, were used for analysis. From each plot, 1 sample tree was selected and cut, after cutting the sample trees diameter and height were measured using stem analysis. The basic data obtained from stem analysis, were transformed into projection format of intervals between time T_1 and T_2 that used to build equation. Mean age, diameter and height were 36 years, 17.9 cm and 13.9 m, respectively. The sample plots were with the gradients of 15-30 degrees, and soil type was moderately moist brown forest soil and mostly loam and clay loam. A summary of relevant plot statistics is given in Table 1.

Table 1: Summary of sample plots statistics

Number of Plots	Mean ages (yr)	Mean DBH (cm)	Mean height (m)	Altitude (m)	Slope (°)	Soil type
20	36	17.9	13.9	100	15-30	B_3

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The methods used for this study were difference equation (Borders *et al.*, 1984) that has been used widely for growth and yield modeling studies. The main standard statistical procedures used were non-linear least-squares regression based on PROC NLIN in Statistical Analysis System (SAS Inc, 1990). Among the algorithms of PROC NLIN procedures used to estimate parameters, the derivative-free method (DUD) that was found to be best in convergence, was adopted for non-linear least-squares regression (Ralston and Jennrich, 1979).

The PROC UNIVARIATE procedure was also used to examine the residuals and provide several statistics that are valuable for making inferences about residual patterns. The important values utilized in the analysis of this study were such as mean of residuals, skewness, kurtosis and extreme values. In addition, graphical charts and plots were used to check the distributions of residuals with regard to normality of errors. Residual errors were plotted against predicted values to determine goodness of fit. Because whether or not the residual patterns lay normally about the zero references line was the important criterion for judging the independent distribution.

The commonly adopted projection equations are log-reciprocal (Schumacher, 1939; Woollons and Wood, 1992), Chapman-Richards (Piennar and Turnbull, 1973; Goulding, 1979), Gompertz (Whyte and Woollons, 1990), Weibull (Yang *et al.*, 1978; Goulding and Shiley, 1979) and Hossfeld (Liu Xu, 1990). There are two types of projection functions used for tree growth models, namely anamorphic and polymorphic functions. Firstly, several frequently used and their accuracy of estimation proved anamorphic equations were assayed such as Schumacher, Chapman-Richards, Hossfeld and Gompertz functions. Then, polymorphic forms of Schumacher, Chapman-Richards, Hossfeld and Gompertz equations were fitted to the data. The functional forms of projection equations used are presented in Table 2.

Table 2: General form of projection equations applied to data

Equation name	Equation Forms*
Schumacher anamorphic	$Y_2 = Y_1 \exp(-\beta (1/T_1^{\gamma} - 1/T_2^{\gamma}))$
Hossfeld anamorphic	$Y_2 = 1/((1/Y_1) + \beta(1/T_2^{\gamma} - 1/T_1^{\gamma}))$
Chapman-Richards Anamorphic	$Y_2 = Y_1((1-\exp(-\beta T_2)) / (1-\exp(-\beta T_1)))^{\gamma}$
Gompertz anamorphic	$Y_2 = Y_1 \exp(-\beta(\exp(\gamma T_2)-\exp(\gamma T_1)))$
Schumacher polymorphic	$Y_2 = \exp(\ln(Y_1)(T_1/T_2)^{\beta} + \alpha(1 - (T_1/T_2)^{\beta}))$
Chapman-Richards poly	$Y_2 = (\alpha/\gamma)^{[1/(1-\beta)]}(1 - (1 - (\gamma/\alpha)Y_1^{(1-\beta)})$ $\exp(-\gamma(1-\beta)(T_2 - T_1))^{[1/(1-\beta)]}$
Gompertz polymorphic	$Y_2 = \exp(\ln(Y_1)\exp(-\beta(T_2-T_1) + \gamma(T_2^2-T_1^2)$ $+ \alpha(1- \exp(-\beta(T_2-T_1) + \gamma(T_2^2-T_1^2)))))$
Hossfeld polymorphic	$Y_2 = 1/((1/Y_1)(T_1/T_2)^{\gamma} + (1/\alpha)(1 - (T_1/T_2)^{\gamma}))$

* Y_1 = Diameter and height of trees at age T_1
Y_2 = Diameter and height of trees at age T_2
Exp = exponential function

3 Results and Discussions

3.1 Prediction of diameter growth

Most anamorphic equations generally produced biased residuals patterns, though Schumacher anamorphic function proved little bit superior in statistics of residuals and residuals patterns to other anamorphic functions. The statistics of residuals of the anamorphic equations fitted are presented in Table 3 with corresponding mean square error values (MSE).

Table 3: Statistics of residuals with the anamorphic equations fitted to data

Equation name	MSE	Mean of residuals	Skewness	Kurtosis
Schumacher	1.688	0.303	-0.212	-0.617
Chapman-Richards	1.829	0.301	-0.031	-0.587
Hossfeld	3.251	5.08	0.58	0.32
Gompertz	2.102	0.792	0.177	0.233

Then, polymorphic forms of Schumacher, Chapman-Richard, Hossfeld and Gompertz equations were fitted to the data. Most of the polymorphic equations generally fitted well without apparent bias in residuals pattern and showed better fit than anamorphic forms of equations. In the Chapman-Richards equation, the confidence interval of the coefficients of γ was not significant $\alpha = 0.05$. Comparing residual pattern and mean square error values, the Schumacher polymorphic function, equation (1), with mean square error (MSE) 1.11 was found to represent better than the other equation. The fitted coefficients and mean square error are shown in Table 4.

$$D_2=\exp(\ln(D_1)(T_1/T_2)^{\beta}+\alpha(1-(T_1/T_2)^{\beta})) \qquad (1)$$

Table 4: Coefficients for polymorphic equation fitted to data

Model Name	Coefficients α	Coefficients β	Coefficients γ	MSE
Schumacher	3.639	0.673	-	1.111
Chapman-Richards	2.305	-0.024	0.000	2.654
Gompertz	3.194	0.098	0.0008	1.520
Hossfeld	26.691	-	1.523	1.424

A plot of residual values against predicted values is given in Figure 1. A plotting of residuals against predicted values indicated that a random pattern around zero with little biased trend. PROC UNIVARIATE in SAS showed that residual statistics were satisfactory as it contained -0.113 value for skewness and -1.202 value for kurtosis. The skeweness and kurtosis of a normal distribution is zero, but in practice values of these lesser or greater than zero result from least-square regression. A Shapiro-Wilk test for normality was totally accepted as 0.95 that is very closed to 1 of normal distribution.

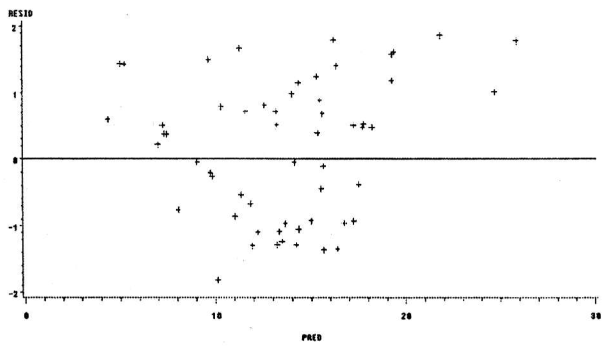

Figure1: A plot of residuals against the predicted for diameter polymorphic projection equation

3.2 Prediction of height growth

The anamorphic and polymorphic functions were applied, such as the log-reciprocal equation, Chapman-Richards, Gompertz and Hossfeld. The fitted coefficients and mean square errors are shown in Tables 5 and 6.

Table 5: Coefficients for anamorphic equation fitted to data

Model Name	Coefficients			MSE
	α	β	γ	
Schumacher	0.131	8.599	-	0.599
Chapman-Richards	-	0.014	-0.899	0.601
Hossfeld	-	0.677	0.589	1.266
Gompertz	-	0.077	-2.624	0.798

Table 6: Coefficients for polymorphic equation fitted to data

Model Name	Coefficients			MSE
	α	β	γ	
Schumacher	4.282	0.359	-	0.458
Chapman-Richards	-	-0.024	-	-
Gompertz	3.120	0.072	0.0005	0.520
Hossfeld	29.517	-	1.129	0.514

Most anamorphic equations showed to be unsuitable with residual patterns, while the Schumacher equation had the lowest mean square errors (MSE) value, which has been used as first option for selecting the best fitting model because the equation with the least biased residuals patterns has been found to have the lowest MSE values, among the anamorphic equations.

None of the asymptotic 95% confidence intervals of each coefficient contained zero that means the coefficients are significant at the α= 0.05 level. However, coefficients α, β and γ of the Chapman-Richards polymorphic equation were failed to converge. Therefore, the Schumacher polymorphic function, equation (2), that has the lowest MSE (0.458) value was found to represent the best fit.

$$H_2=\exp(\ln(H_1)(T_1/T_2)^{\beta}+\alpha(1-(T_1/T_2)^{\beta})) \qquad (2)$$

The data were evidently well balanced with no apparent bias or systematic patterns and showed goodness of fit as shown in Figure 2.

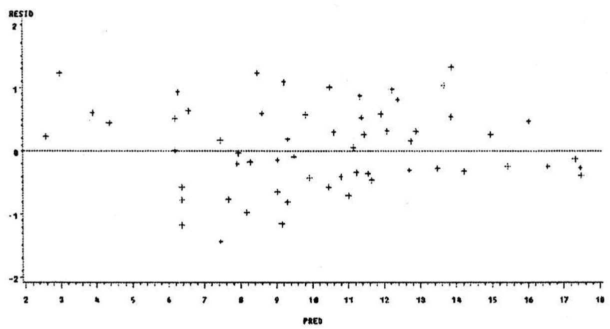

Figure 2: A plot of residuals against the predicted for height polymorphic projection equation

The equation (2) contained desirable functions commonly used in growth and yield models, such as compatibility, consistency, and path-invariance (Clutter *et al.*, 1983). As T_2 approaches infinitely, H_2 approaches the upper asymptote α, when T_1 equals T_2 then H_1 equals H_2 (consistency property), and the projection from T_1 to T_3 yields the same result as the projection from T_1 to T_2 followed by projection from T_2 to T_3 (path-invariance property).

The PROC UNIVARIATE statistics in Table 7 proved that the equation provides an unbiased precise estimate of height as it contained -0.036 value for skewness which indicated little bit long tails to the right and -0.615 value for kurtosis, the heaviness of tails in a distribution. A Shapiro – Wilk test for normality was totally accepted as 0.970 that is very closed to 1 of

Table 7: Summary of statistics of residual values for height projection equation

Statistics Name	Values
Mean	0.086
Skewness	-0.036
Kurtosis	-0.615
W: Normal	0.970

normal distribution. The mean of the average residuals was 0.01 m, which represents a slight underestimation, but showed very accurate and precise estimation. The equation gave a maximum residual of 1.3m, a minimum residual of -1.4 m, and 95% of residuals lay ±1.4 m.

3.3 Deriving site index equation and curves

The site index equation (3) can be derived from height equation (2) by setting H_2 =site index (SI) when $T_2 = 40$ years, which is used for the base age of *Pinus thunbergii* in this study because it is closer to the rotation age.

$$SI=\exp(\ln(H_1)(T_1/40)^{\gamma}+\alpha(1-(T_1/40)^{\gamma})) \qquad (3)$$

Where, α= 4.2825, γ= 0.3599.

Site index curves can then be generated by rearranging equation (3), and making H_1 the subject. Substituting SI with any required site index values (e.g. 10, 20, 30, and 40) results in polymorphic height growth curves. Figure 3 shows a set of site index curves resulted from equation (4).

$$H_1 = \left[\frac{SI}{\exp(\alpha(1-(T_1/40)^{\beta}))}\right]^{1/(T_1/40)^{\beta}} \qquad (4)$$

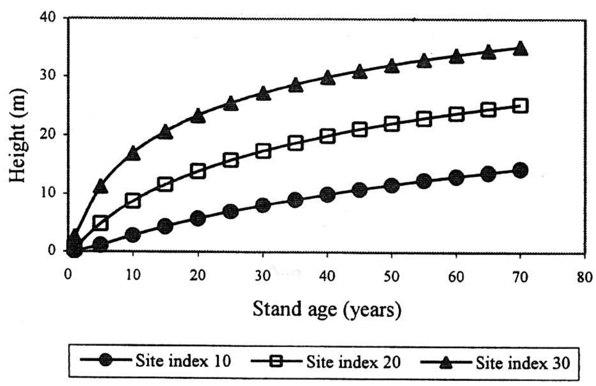

Figure 3: Site index curves for *Pinus thunbergii* derived from a height equation

4 Conclusions

The Schumacher polymorphic equation provided satisfactory models of the diameter and site index equation for *Pinus thunbergii* grown Saemangeum surrounding regions in Korea. This was ensured by comparing the respective residual mean squares values, where the Schumacher polymorphic equation was the lowest in value, as well as better residual patterns and residual statistics.

It is unrealistic to expect a unique function to perform consistently better than others with forest growth data. However, the initial selection of appropriate equations is most important for success of the goodness of fit models. This research will provide basic information of growth pattern for *Pinus thunbergii*, which is expected to use as main species for windbreak stands after finishing 'Saemangeum' reclamation work.

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